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SIGNAL AVERAGING OF NON-STATIONARY NOISE

Jeroen R. Willemsen, André C. Linnenbank, Mark Potse, and Cornelis A. Grimbergen

Medical Physics Department, Academic Medical Center, P.O. Box 22700, 1100 DE Amsterdam, The Netherlands, e-mail: A.C.Linnenbank@amc.uva.nl

***Abstract*—A signal averaging method is described that takes into account the non-stationarity of the noise in the averaged signals in order to obtain the optimal noise reduction.**

Introduction

In coherent averaging of electrocardiograms the noise is assumed stationary. In practice, however, the noise is often non-stationary. Beats with large noise levels are often discarded. Weighted averaging of beats could take the varying noise levels into account and produce a minimal noise level of the averaged signal.

Methods

The signal model in the case of coherent averaging of non-stationary noise is:

$$y_k(t) = x_k(t) + \sigma_k n(t)$$

where $x_k(t)$ is the signal of interest, which is assumed to have an invariant morphology and to be uncorrelated with $n(t)$. The noise signal $n(t)$ has zero mean and unity variance, and σ_k is the noise level in the kth interval.

If the signal were stationary the variance of the coherent averaged signal would have been σ_k^2/N. If we now define the weighted averaged signal $\overline{y}_{\mathrm{w}}(t)$ as:

$$\overline{y}_{\mathrm{w}}(t) = \sum_{k=1}^{N} w_k y_k(t) \quad \text{with} \quad \sum_{k=1}^{N} w_k = 1$$

Then we can show that the minimal variance is:

$$\overline{\sigma}^2 = \left(\sum_{k=1}^{N} \frac{1}{\sigma_k^2} \right)^{-1} \quad \text{using} \quad w_k = \frac{1}{\sigma_k^2} \cdot \left(\sum_{j=1}^{N} \frac{1}{\sigma_j^2} \right)^{-1}$$

Results

We have simulated a non-stationary noise added to an artificial ECG-complex. For this simulation noise levels randomly chosen from a uniform distribution between 3 and 10 $\mu\mathrm{V_{rms}}$ were used. In the upper panel of figure 1 the resulting noise levels as a function of the number of averaged signals are shown. In the lower panel the noise levels used for the simulation are shown. The noise levels after 32 signals averaged are 0.97 $\mu\mathrm{V_{rms}}$ for the weighted average and 1.17 $\mu\mathrm{V_{rms}}$ for the unweighted average. In this example, using weighted averaging, 22 complexes would suffice to achieve the same noise level as unweighted averaging with 32 complexes. Note that for the unweighted average the noise actually increased when adding the 3rd complex. The weighted average on the other hand has the property that the noise decreases monotonically when complexes are added.

Conclusions

The use of weighted averaging decreases the number of complexes needed to achieve a predefined noise level. A prerequisite is that the noise levels of all complexes are known. When analyzing data off-line these can easily be measured, but even in signal averaging ECG-devices these levels could be determined on-line.

Acknowledgement

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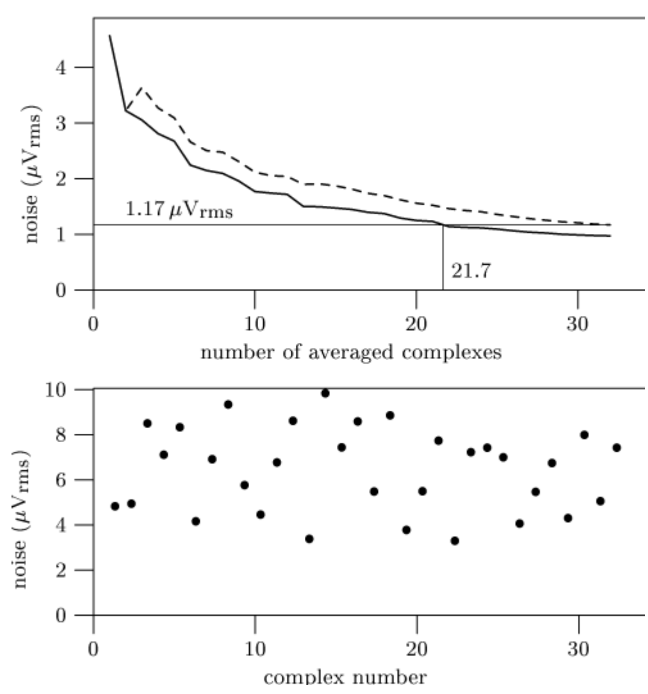

Figure 1. Upper panel shows the noise levels for weighted averaging (solid line) and unweighted averaging (dashed line). The lower panel shows the noise levels of the consecutive complexes.